Two kT/C or not Two kT/C, That is the Question

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Figure 1 shows our discrete integrator (DI) circuit. What is the input referred noise for infinite OTA bandwidth (BW)? How does it change for finite OTA BW?

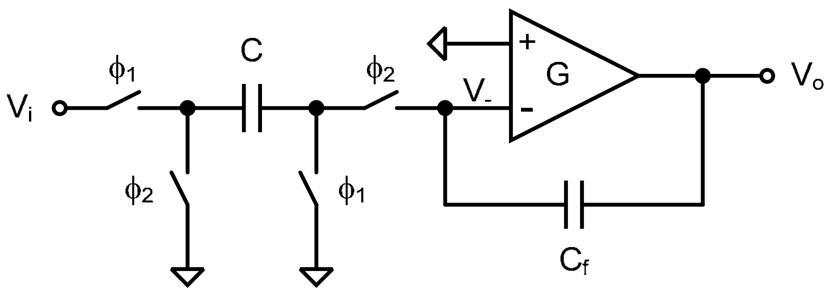

Figure 1. DI circuit. The OTA has transconductance G.

We assume $V_i = 0$ V. At the end of phase 1 the standard kT/C noise is left on the capacitor C:

$$V_{n,1}^2 = \frac{kT}{C} . \tag{1}$$

At the start of phase 2, the corresponding noise charge is taken in at node V_- by the OTA-C_f feedback circuit. Since the noise voltage $V_{n,1}$ was "frozen" on capacitor C after phase 1, this is true even for finite bandwidth, f_0, of the transconductance G, as long as V_o settles within phase 2, which we will assume here.

At the end of phase 2, however, some noise will be left on C again. Call this noise voltage $V_{n,2}$. We do not know what it is for now. The total input referred noise voltage, V_n, is given by:

$$V_n^2 = V_{n,1}^2 + V_{n,2}^2 . \tag{2}$$

In phase 2, it is useful to simplify the OTA-C_f portion of the circuit of Figure 1 as shown in Figure 2.

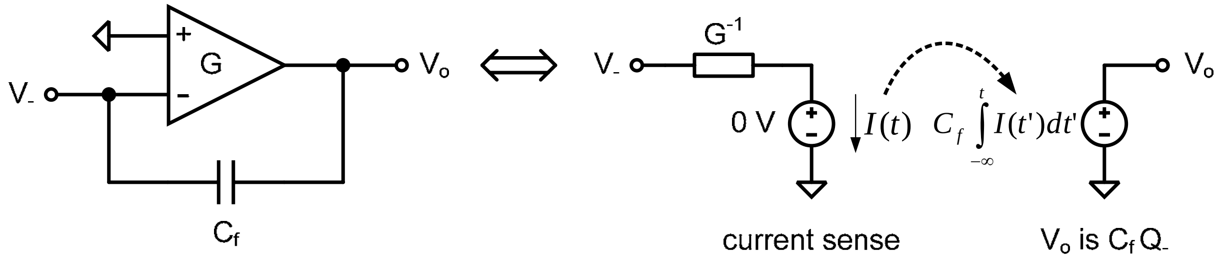

Figure 2. Equivalent circuit of OTA-C_f portion of Figure 1.

The impedance G^{-1} shown on the right-hand side of Figure 2 is easily derived by determining what voltage is required at V_- to give an assumed input current at the same node. The 0-V voltage source merely senses the current entering V_-. This current is integrated to produce an output V_o which is proportional to the total charge Q_- that has passed through V_-. The integration may start at negative infinity or the most recent time the C_f voltage was reset to zero.

Since we are only concerned with input referred noise, only the input section of the simplified circuit on the right-hand side of Figure 2 matters to us. It allows us to make a simple model of the noise contributors during phase 2, see Figure 3.

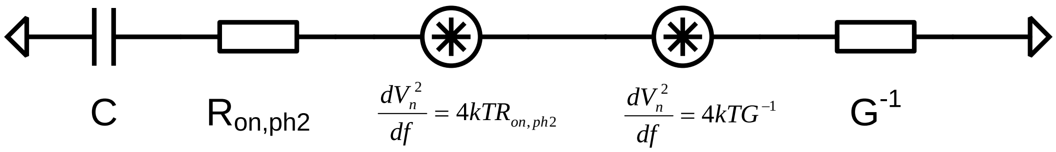

Figure 3. Equivalent circuit to determine noise on capacitor C during phase 2. $R_{on,ph2}$ is the sum of the phase 2 on-switch resistances.

Now we can determine $V_{n,2}$. First, for infinite BW OTA, G^{-1} behaves just like an infinite BW resistor (such as $R_{on,ph2}$). The textbook RC integration argument applies and we have:

Infinite f_0:
$$V_{n,2}^2 = \frac{kT}{C} \,. \tag{3}$$

And with Equation (2):

Infinite f_0:
$$\boxed{V_n^2 = \frac{2kT}{C}} \tag{4}$$

If the OTA has a finite bandwidth f_0, things change. For simplicity assume that $G = G_0$ up to f_0 and $G = 0$ for frequencies beyond f_0. Using Figure 3 again to integrate over all frequencies, we find:

Any f_0:
$$V_{n,2}^2 = 4kTR_{total}\int_0^{f_0}\frac{1}{1+(2\pi fR_{total}C)^2}df + \lim_{G\to 0} 4kTG^{-1}\int_{f_0}^{\infty}\frac{1}{1+(2\pi fG^{-1}C)^2}df \,, \tag{5}$$

where:

$$R_{total} = R_{on,ph2} + G_0^{-1} \,. \tag{6}$$

For a fixed f_0 and G approaching zero, or G^{-1} to infinity, the second integral becomes zero. The first integral, as is well known, leads to:

Any f_0:
$$V_{n,2}^2 = \frac{4kT}{2\pi C}\arctan(2\pi f_0 R_{total}C) \,. \tag{7}$$

This is smaller than kT/C whenever f_0 is finite. Neglecting the switch on resistances for a moment, (7) is noticeably less than kT/C when $f_0 < G_0/(2\pi C)$. In other words, when the OTA BW is smaller than the closed loop limited BW. In general we conclude from (1,2,7):

Finite f_0:

$$\boxed{V_n^2 < \frac{2kT}{C}} \tag{8}$$

Equations (4) and (8) answer our questions.

In summary: Limited DI OTA bandwidth makes the integrator load look like an open circuit rather than a finite resistance beyond that same bandwidth. For a conventional fixed resistor the reduced BW is compensated by an increase in noise power (V^2/Hz) in the frequency integration. Here only the tail of the integration is affected and the reduced BW leads to a quadratic decrease with increased effective resistance, which more than offsets the first order noise power increase. As a result, finite OTA BW changes the resistor thermal noise contribution from the infinite BW result 2kT/C to a smaller value.